

average would be -0.50 , for which the antilogarithm is decimal 0.32 or a Snellen notation of $20/63$ (point C in the Figure).

In our example, if we were to calculate the inappropriate arithmetic mean using the decimal values, as suggested most recently by Vila-Coro and Vila-Coro,³ we would obtain an average decimal value of 0.55 ($[0.1 + 1.0]/2$) or $20/36$ (point D in the Figure). This method overestimates the true geometric mean visual acuity and minimizes the contribution of the poor visual acuity samples.

A second incorrect method is to take the arithmetic mean of the minimum angle of resolution, which is equivalent to taking the average of the denominators of the Snellen notation. Pincus⁴ used this method to determine the average visual acuity for a given refractive error. Using this incorrect method in our previous example, the mean visual acuity would have been $20/110$ ($[200 + 20]/2$). This arithmetic method will severely underestimate the actual geometric mean visual acuity (point E in the Figure). If one takes the geometric mean of the Snellen denominators, the minimum visual angles of resolution, or the Snellen fractions or decimals, the result is the same, $20/63$, which is the correct result.

Fortunately, with the newer visual acuity charts (for example, Bailey-Lovie, ETDRS, PERK) that have an equal number of letters on a line and a constant geometric progression between lines, the actual line numbers are directly proportional to the logarithm of the visual acuity, as we have shown previously.⁵ When these charts are used, the geometric mean visual acuity is more simply obtained by calculating the arithmetic mean of the number of lines or letters correct, then converting the result to the corresponding Snellen visual acuity.⁵ With many standard projector charts, however, in which the number of letters on each line are not equal or the progression is not consistent (for example, the $20/125$ and $20/160$ lines are missing and a $20/70$ line should not be present), this simple method may not be used and the logarithmic method is necessary.

Likewise, when other statistical analyses are performed on visual acuity, such as correlation coefficients or standard deviations, they must be calculated using the logarithm of the visual acuity or logarithm of the visual angle as Sloan⁶ has shown in her correlations of visual acuity with refractive error. Care should be taken by authors and reviewers to assure that these principles are followed so that mean visual acuity

presented in a study is valid and comparable to other studies. Unfortunately, many of the mean visual acuities in published reports have used one of the two incorrect arithmetic methods, which make most comparisons of mean visual acuity invalid.

References

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Mean Visual Acuity

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Calculating mean visual acuity on a series of patients has been done incorrectly in most studies, which leads to a significant overestimate or underestimate of the true mean visual acuity. The basic problem relates to the difference between the arithmetic and geometric mean of a set of numbers. For the correct mean visual acuity, calculating the geometric mean yields the proper value.

Modern visual acuity charts are designed so that the letter sizes on the chart follow a geometric progression (that is, advance in uniform steps on a logarithmic scale).¹ The International Council of Ophthalmology Committee on optotypes accepted the original recommendation of Green² to have the letter sizes change by 0.1-log unit steps, which is equivalent to letter sizes changing by a factor of 1.2589 between lines.¹ This standard led to the LogMAR (logarithm of the minimum angle of resolution) notation¹ as plotted in the Figure.

The letter sizes between 20/10 and 20/200 progress in a linear fashion on a logarithm scale, and visual performance midway between

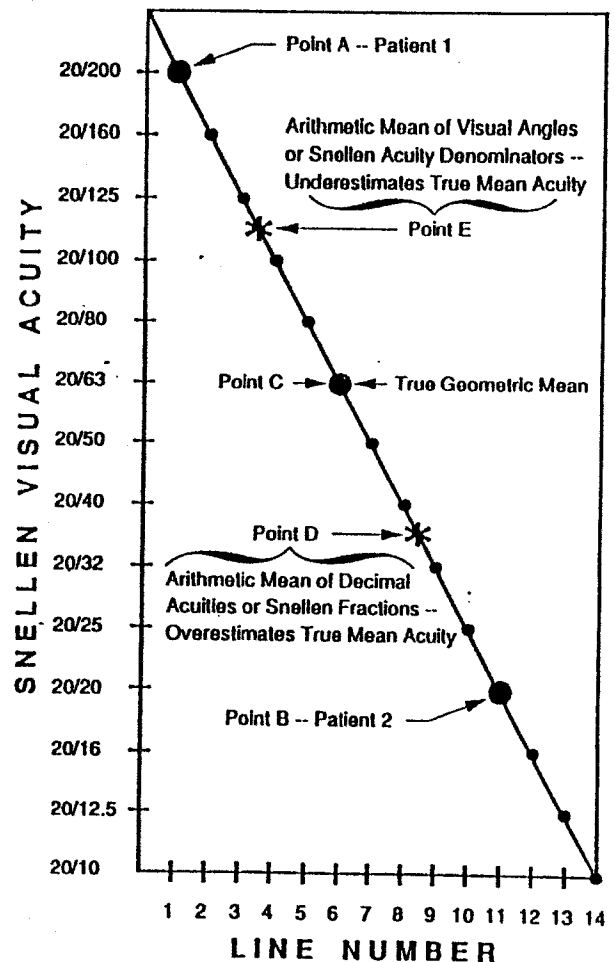


Figure (Holladay and Prager). The true geometric mean visual acuity between Patient 1 with visual acuity of 20/200 (Point A) and Patient 2 with visual acuity of 20/20 (Point B), is 20/63 (Point C). The incorrect arithmetic mean obtained by taking the average of the decimal visual acuities or Snellen fractions is 20/36 (Point D). The arithmetic mean obtained by taking the average of the Snellen visual acuity denominators or visual angles is 20/110 (Point E). Arithmetic means severely overestimate or underestimate the true geometric mean visual acuity.

20/200 (line 1) and 20/20 (line 11) is 20/63 (line 6) (Figure). This visual acuity of 20/63 is the geometric mean of these two visual acuities. Mathematically, the geometric mean is calculated by taking the logarithm of each of the sample values, determining the average of the logarithm values, then taking the antilogarithm of this average. In this example, the logarithm of 20/200 (point A in the Figure) is -1.0 and the log of 20/20 (point B in the Figure) is 0 . The